Mr. Michael T. Davis WLPCS Calculus	Sections 5.2 & 5.3 In-Class Problem Solving April 6, 2018

Name: Solutions / Answers

1. Given the function $f(x) = xe^x$, use the first derivative and the second derivative to determine:

a. The interval(s) on which the function is increasing,

$f'(x) = e^x + xe^x = e^x(1+x)$ $\quad 0 = e^x(1+x)$

$e^x \neq 0 \quad 1+x=0 \quad x=-1$ c.P.

$- - - \quad + + +$ f' (sign line, critical point at -1)

$f(x)$ is increasing on $[-1,\infty)$ since $f'(x)>0$

b. The interval(s) on which the function is decreasing,

referring to the sign analysis above, $f(x)$ is decreasing on $(-\infty,-1]$ since $f'(x)<0$

c. The x value of any point of inflection,

$f''(x) = e^x + e^x + xe^x = 2e^x + xe^x = e^x(2+x)$

$- - - \quad + + +$ f'' (sign line, at -2)

$0 = e^x(2+x) \quad e^x \neq 0 \quad 2+x=0 \quad x=-2$

$f(x)$ has a P. of I. at $x=-2$ since $f''(-2)=0$ and f'' changes sign at $x=-2$.

d. The interval(s) on which the graph is concave up,

Referring to the sign analysis for f'' above, $f(x)$ is concave up on $(-2,\infty)$ since $f''(x)>0$

e. The interval(s) on which the graph is concave down.

Referring to the sign analysis for f'' above, $f(x)$ is concave down on $(-\infty,-2)$ since $f''(x)<0$

f. Use the second derivative test to identify any minimum or maximum values.

$x=-1$ is a C.P. $\quad f(-1) = -e^{-1} = \frac{-1}{e}$

$f(x)$ has an abs. min of $y=\frac{-1}{e}$ at $x=-1$ since $f''(-1)>0$ in which case the graph of $f(x)$ is concave up at $x=-1$.

2. Given the function $f(x) = 8x - x^2$, use the first derivative and the second derivative to determine:

a. The interval(s) on which the function is increasing,

$f'(x) = 8 - 2x \quad 0 = 8 - 2x \quad x = 4$ c.P.

$+ + + \quad - - -$ f'

4

$f(x)$ is increasing on $(-\infty, 4]$ since $f'(x) > 0$ on $(-\infty, 4)$

b. The interval(s) on which the function is decreasing,

Referring to a. above, $f(x)$ is decreasing on $[4, \infty)$ since $f'(x) < 0$ on $(4, \infty)$

c. The x value of any point of inflection,

$f''(x) = -2 \quad f''(x) < 0$ for all real x

$f''(x)$ does not change sign; therefore, $f(x)$ is always concave down; therefore, $f(x)$ does not have a P. of I.

d. The interval(s) on which the graph is concave up,

$- - - - - -$ f''

$f(x)$ is never concave up since $f''(x) < 0$

e. The interval(s) on which the graph is concave down,

Referring to d. above, $f(x)$ is concave down on $(-\infty, \infty)$ since $f''(x) < 0$

f. Use the second derivative test to identify any minimum or maximum values.

$f(4) = 16$

$x = 4$ is a c.P. $\quad f''(4) < 0 \quad f(x)$ is concave down

$f(x)$ has an abs. max of $y = 16$ at $x = 4$ since $f'(4) = 0$ and $f''(4) < 0$.

3. Given the function $f(x)=x^3+\frac{9}{2}x^2-12x-2$, use the first derivative and the second derivative to determine:

a. The interval(s) on which the function is increasing, $f'(x) = 3x^2+9x-12$

$0 = 3(x^2+3x-4) = 3(x+4)(x-1)$ $\quad x=-4$ c.p. $\quad x=1$ c.p.

+ + + $\quad$ − − − $\quad$ + + + $\quad f'$

$-4 \quad 1$

$f(x)$ is increasing on $(-\infty,-4] \cup [1,\infty)$ since $f'(x)>0$

b. The interval(s) on which the function is decreasing,

Referring to a. above,
$f(x)$ is decreasing on $[-4,1]$ since $f'(x)<0$

c. The x value of any point of inflection,

$f''(x) = 6x+9 \quad 0=6x+9 \quad x=\frac{-3}{2}$ c.p.

− − − $\quad$ + + + $\quad f''$

$\frac{-3}{2}$

$f(x)$ has a P. of I at $x=-\frac{3}{2}$ since $f''(\frac{-3}{2})=0$ and f'' changes sign at $x=-\frac{3}{2}$

d. The interval(s) on which the graph is concave up,

Referring to c. above,
$f(x)$ is concave up on $(-\frac{3}{2},\infty)$ since $f''(x)>0$ on $(-\frac{3}{2},\infty)$

e. The interval(s) on which the graph is concave down,

Referring to c. above,
$f(x)$ is concave down on $(-\infty,\frac{-3}{2})$ since $f''(x)<0$ on $(-\infty,\frac{-3}{2})$

f. Use the second derivative test to identify any minimum or maximum values.

$f(-4) = -64+72+48-2 = 54$

$f(1) = 1+\frac{9}{2}-12-2 = -8.5$

$f(x)$ has a local min of $y=-8.5$ at $x=1$ since $f''(1)>0$

$f(x)$ has a local max of $y=54$ at $x=-4$ since $f''(-4)<0$

4. Given the function $f(x)=x^4-6x^2+12$, use the first derivative and the second derivative to determine:

a. The interval(s) on which the function is increasing, $f'(x)=4x^3-12x$

$0=4x(x^2-3)$ $\quad x=0$ C.P. $\quad x=\sqrt{3}$ C.P. $\quad x=-\sqrt{3}$ C.P.

$- - - \quad + + + \quad - - - \quad + + +$ f'

$-\sqrt{3} \quad 0 \quad \sqrt{3}$

$f(x)$ is increasing on $[-\sqrt{3},0] \cup [\sqrt{3},0)$ since $f'(x)>0$

b. The interval(s) on which the function is decreasing,

Referring to a. above,

$f(x)$ is decreasing on $(-\infty,-\sqrt{3}] \cup [0,\sqrt{3}]$ since $f'(x)<0$

c. The x value of any point of inflection,

$f''(x)=12x^2-12=12(x-1)(x+1)$ $\quad 0=12(x-1)(x+1)$

$x=1 \quad x=-1$

$+ + + \quad - - - \quad + + +$ f''

$-1 \quad 1$

$f(x)$ has P. of I at $x=-1$ and at $x=1$ since $f''(-1)=0$ and $f''(1)=0$ and $f''(x)$ changes sign at $x=-1$ and $x=1$.

d. The interval(s) on which the graph is concave up,

Referring to c. above,

$f(x)$ is concave up on $(-\infty,-1) \cup (1,\infty)$ since $f''(x)>0$

e. The interval(s) on which the graph is concave down,

Referring to c. above,

$f(x)$ is concave down on $(-1,1)$ since $f''(x)<0$

f. Use the second derivative test to identify any minimum or maximum values.

$f(-\sqrt{3})=3 \quad f(0)=12 \quad f(\sqrt{3})=3$

$f(x)$ has an abs. min. of $y=3$ at $x=-\sqrt{3}$ and $x=\sqrt{3}$ since $f'(-\sqrt{3})=0$ and $f''(-\sqrt{3})>0$ and since $f'(\sqrt{3})=0$ and $f''(\sqrt{3})>0$.

$f(x)$ has a local max of $y=12$ at $x=0$ since $f'(0)=0$ and $f''(0)<0$